



Math Weekly Lesson Preparation Guide

Teacher Name: Kimberly West	Grade: 11 th /12 th Precalculus
Week of: February 24 th thru 28 th	Unit: 5 Lesson Numbers: 4.8 Solving Problems with Trigonometry 5.1 Fundamental Identities 5.2 Proving Trigonometric Identities

Purpose: The Weekly Lesson Preparation Guide is to provide a structure that encourages teachers to think through and internalize the daily/weekly instructional expectations.

Planning Questions	Monday Lesson 4.8	Tuesday Lesson 5.1	Wednesday Lesson 5.2	Thursday Lesson 5.2	Assessment OR Remediation
1. Which specific Tennessee standard(s) are being addressed in this lesson? What is the focus of this lesson? What will the lesson objective be for each day?	<p>P.G.AT.A.1 Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles.</p> <p>Objective: I can determine the angle of elevation/depression and distance using right triangles.</p>	<p>P.G.TI.A.1 Apply trigonometric identities to verify identities and solve equations. Identities include : Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle.</p> <p>Objective: I can simplify trigonometric expressions using basic trigonometric identities.</p>	<p>P.G.TI.A.1 Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle.</p> <p>Objective: I can prove an algebraic or trigonometric identity.</p>	<p>P.G.TI.A.1 Apply trigonometric identities to verify identities and solve equations. Identities include : Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle.</p> <p>Objective: I can prove an algebraic or trigonometric identity.</p>	<p>P.G.AT.A.1 Use the definitions of the six trigonometric ratios as ratios of sides in a right triangle to solve problems about lengths of sides and measures of angles.</p> <p>P.G.TI.A.1 Apply trigonometric identities to verify identities and solve equations. Identities include: Pythagorean, reciprocal, quotient, sum/difference, double-angle, and half-angle.</p>

Adapted from TDOE Unit and Lesson Preparation Guides

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Modeling:

2. **Complete all tasks** included in the lesson and review the sample/anticipated student responses. For each task consider:

- What are the multiple solution paths students might take to solve this problem?
- What is the purpose of this task? Specifically, which aspect(s) of rigor are being addressed (conceptual understanding, procedural fluency, and/or application)? How does this differ based on the solution path
- Given this purpose, what key concepts and vocabulary might students

Chapter 4
Trigonometric Functions
Section 4.8
Solving Problems with Trigonometry

What you'll learn about

- More Right Triangle Problems
- Simple Harmonic Motion

... and why

These problems illustrate some of the better-known applications of trigonometry.

Angle of Elevation, Angle of Depression

An **angle of elevation** is the angle through which the eye moves up from horizontal to look at something above. An **angle of depression** is the angle through which the eye moves down from horizontal to look at something below.

Example 1: Using Angle of Elevation

The angle of elevation from the buoy to the top of the Barnegat Bay lighthouse 130 feet above the surface of the water is 5° . Find the distance x from the base of the lighthouse to the buoy.

$$\tan 5^\circ = \frac{130}{x}$$

$$x = \frac{130}{\tan 5^\circ} \approx 1485.9$$

The buoy is about 1486 feet from the base of the lighthouse.

Chapter 5
Analytic Trigonometry
Section 5.1
Fundamental Identities

What you'll learn about

- Identities
- Basic Trigonometric Identities
- Pythagorean Identities
- Cofunction Identities
- Odd-Even Identities
- Simplifying Trigonometric Expressions
- Solving Trigonometric Equations

... and why

Identities are important when working with trigonometric functions in calculus.

Basic Trigonometric Identities

Reciprocal Identities

$$\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\sin \theta = \frac{1}{\csc \theta} \quad \cos \theta = \frac{1}{\sec \theta} \quad \tan \theta = \frac{1}{\cot \theta}$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities
$$\cos^2 \theta + \sin^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

Chapter 5
Analytic Trigonometry
Section 5.2
Proving Trigonometric Identities

What you'll learn about

- A Proof Strategy
- Proving Identities
- Disproving Non-Identities
- Identities in Calculus

... and why

Proving identities gives you excellent insights into the way mathematical proofs are constructed.

General Strategies I for Proving an Identity

1. The proof begins with the expression on one side of the identity.
2. The proof ends with the expression on the other side.
3. The proof in between consists of showing a sequence of expressions, each one easily seen to be equivalent to its preceding expression.

General Strategies II for Proving an Identity

1. Begin with the more complicated expression and work toward the less complicated expression.
2. If no other move suggests itself, convert the entire expression to one involving sines and cosines.
3. Combine fractions by combining them over a common denominator.

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Vocabulary

- Angle of Elevation
- Angle of Depression
- Solve Algebraically
- Knots
- Simple Harmonic Motion
- Frequency
- Identities
- Domain of Validity
- Reduction Formula

need to understand to access the task?

Example 2: Making Indirect Measurements (1 of 2)

From the top of a 100 ft. tall air traffic control tower, an airplane is observed flying toward the tower. If the angle of elevation of the airplane changes from 32° to 10° during the period of observation, and the altitude of the airplane changes from 500 ft to 200 ft, how far does the plane travel?

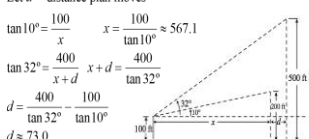
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Example 2: Making Indirect Measurements (2 of 2)

The figure models the situation.

Let x = distance from the tower at the second observation.

Let d = distance plane moves



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Example 1: Using Identities

Find $\sin \theta$ and $\cos \theta$ if $\tan \theta = 3$ and $\cos \theta < 0$.

$$\begin{aligned}
 1 + \tan^2 \theta &= \sec^2 \theta & \text{To find } \sin \theta, \text{ use } \tan \theta = 3 \\
 1 + 9 &= \sec^2 \theta & \text{and } \cos \theta = -1/\sqrt{10}. \\
 \sec \theta &= \pm \sqrt{10} & \tan \theta = \frac{\sin \theta}{\cos \theta} \\
 \cos \theta &= -1/\sqrt{10} & \sin \theta = \cos \theta \tan \theta \\
 & & \sin \theta = \left(-1/\sqrt{10}\right)(3) \\
 & & \sin \theta = -3/\sqrt{10}
 \end{aligned}$$

Therefore, $\cos \theta = -1/\sqrt{10}$ and $\sin \theta = -3/\sqrt{10}$

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Cofunction Identities

Angle A: $\sin A = \frac{y}{r}$, $\tan A = \frac{y}{x}$, $\sec A = \frac{r}{x}$
 $\cos A = \frac{x}{r}$, $\cot A = \frac{x}{y}$, $\csc A = \frac{r}{y}$
 Angle B: $\sin B = \frac{x}{r}$, $\tan B = \frac{x}{y}$, $\sec B = \frac{r}{y}$
 $\cos B = \frac{y}{r}$, $\cot B = \frac{y}{x}$, $\csc B = \frac{r}{x}$

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Cofunction Identities

$$\begin{aligned}
 \sin\left(\frac{\pi}{2} - \theta\right) &= \cos \theta & \cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta \\
 \tan\left(\frac{\pi}{2} - \theta\right) &= \cot \theta & \cot\left(\frac{\pi}{2} - \theta\right) &= \tan \theta \\
 \sec\left(\frac{\pi}{2} - \theta\right) &= \csc \theta & \csc\left(\frac{\pi}{2} - \theta\right) &= \sec \theta
 \end{aligned}$$

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Even-Odd Identities

$$\begin{aligned}
 \sin(-x) &= -\sin x & \cos(-x) &= \cos x & \tan(-x) &= -\tan x \\
 \csc(-x) &= -\csc x & \sec(-x) &= \sec x & \cot(-x) &= -\cot x
 \end{aligned}$$

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General Strategies III for Proving an Identity

1. Use the algebraic identity $(a + b)(a - b) = a^2 - b^2$ to set up applications of the Pythagorean identities.
2. Always be mindful of the "target" expression, and favor manipulations that bring you closer to your goal.

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Example: Setting up a Difference of Squares

Prove the identity: $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$.

$$\begin{aligned}
 \frac{\sin x}{1 - \cos x} &= \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} \\
 &= \frac{(\sin x)(1 + \cos x)}{1 - \cos^2 x} \\
 &= \frac{(\sin x)(1 + \cos x)}{\sin^2 x} \\
 &= \frac{1 + \cos x}{\sin x}
 \end{aligned}$$

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Identities in Calculus

1. $\cos^3 x = (1 - \sin^2 x)(\cos x)$
2. $\sec^4 x = (1 + \tan^2 x)(\sec^2 x)$
3. $\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$
4. $\cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x$
5. $\sin^3 x = (1 - 2\cos^2 x + \cos^4 x)(\sin x)$
6. $\sin^2 x \cos^3 x = (\sin^2 - 2\sin^2 x + \sin^4 x)(\cos x)$

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Example: Proving an Identity Useful in Calculus

Prove the following identity:

$$\begin{aligned}
 \sin^5 x \cos^3 x &= (\sin x)(\cos^2 - 2\cos^4 x + \cos^6 x) \\
 &= (\sin x)(\sin^2 x)^2 (\cos^2 x) \\
 &= (\sin x)(1 - \cos^2 x)^2 (\cos^2 x) \\
 &= (\sin x)(1 - 2\cos^2 x + \cos^4 x)(\cos^2 x) \\
 &= (\sin x)(\cos^2 - 2\cos^4 x + \cos^6 x)
 \end{aligned}$$

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	<p>Vocabulary</p> <ul style="list-style-type: none"> • Angle of Elevation • Angle of Depression • <p>Solve Algebraically</p> <ul style="list-style-type: none"> • Knots • Simple Harmonic Motion • Frequency 	<p>Example: Simplifying by Factoring and Using Identities</p> <p>Simplify the expression $\cos^3 x + \cos x \sin^2 x$.</p> $\begin{aligned}\cos^3 x + \cos x \sin^2 x &= \cos x (\cos^2 x + \sin^2 x) \\ &= \cos x (1) \quad \text{Pythagorean Identity} \\ &= \cos x\end{aligned}$ <p><small>FOURTH GRADE LEARNING Copyright © 2018, 2015, 2013 Pearson Education, Inc. 9</small></p> <p>Example: Simplifying by Expanding and Using Identities</p> <p>Simplify the expression: $\frac{(\csc x - 1)(\csc x + 1)}{\cos^2 x}$</p> $\begin{aligned}\frac{(\csc x - 1)(\csc x + 1)}{\cos^2 x} &= \frac{\csc^2 x - 1}{\cos^2 x} \quad (a + b)(a - b) = a^2 - b^2 \\ &= \frac{\cot^2 x}{\cos^2 x} \quad \text{Pythagorean Identity} \\ &= \frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x} \quad \cot \theta = \frac{\cos \theta}{\sin \theta} \\ &= \frac{1}{\sin^2 x} \\ &= \csc^2 x\end{aligned}$ <p><small>FOURTH GRADE LEARNING Copyright © 2018, 2015, 2013 Pearson Education, Inc. 10</small></p> <p>Example : Solving a Trigonometric Equation (1 of 2)</p> <p>Find all values of x in the interval $[0, 2\pi)$ that solve</p> $\frac{\sin^3 x}{\cos x} = \tan x.$ <p><small>FOURTH GRADE LEARNING Copyright © 2018, 2015, 2013 Pearson Education, Inc. 11</small></p> <p>Example: Solving a Trigonometric Equation (2 of 2)</p> $\begin{aligned}\frac{\sin^3 x}{\cos x} &= \tan x && \text{Reject the possibility that } \cos^2 x = 0 \\ \frac{\sin^3 x}{\cos x} &= \frac{\sin x}{\cos x} && \text{because it would make both} \\ \sin^3 x &= \sin x && \text{sides of the original equation} \\ \sin^3 x - \sin x &= 0 && \text{undefined. } \sin x = 0 \text{ in the interval} \\ \sin x(\sin^2 x - 1) &= 0 && 0 \leq x < 2\pi \text{ when } x = 0 \text{ and } x = \pi. \\ (\sin x)(\cos^2 x) &= 0 \\ \sin x = 0 &\text{ or } \cos^2 x = 0\end{aligned}$ <p><small>FOURTH GRADE LEARNING Copyright © 2018, 2015, 2013 Pearson Education, Inc. 12</small></p>	<p>Vocabulary</p> <ul style="list-style-type: none"> • Reduction Formula 	<p>Vocabulary</p> <ul style="list-style-type: none"> • Reduction Formula 	
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		Vocabulary <ul style="list-style-type: none"> • Identities • Domain of Validity 			
3. What specific tasks/problems will you use to reveal understanding of the grade-level standard(s)? (refer to the Instructional Focus Document Evidence of Learning Statements)	*Selective Practice Problems from pages 403-404 *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	*Selective Practice Problems from pages 403-404 *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	*Selective Practice Problems from pages 411-412 *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	*Selective Practice Problems from pages 411-412 *Look and listen for proper steps and vocabulary used to explain each step in the problem solving process	
Additional Considerations					
If your lesson contains homework, how will you utilize the work? Will you need to send scaffolding notes home? Is there a strategy you can use to maximize homework?		Homework will be utilized by: Align with Learning Objectives: Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. Variety of Tasks: Include different types of problems (e.g., practice, application, extension) to cater to various levels of understanding and	Homework will be utilized by: Align with Learning Objectives: Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. Variety of Tasks: Include different types of problems (e.g., practice, application, extension) to cater to various levels of	Homework will be utilized by: Align with Learning Objectives: Ensure that homework directly relates to the concepts taught in class, allowing students to apply their learning. Variety of Tasks: Include different types of problems (e.g., practice, application, extension) to cater to various levels of	

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		<p>to reinforce the concept from multiple angles.</p> <p>Scaffolded Problems: Start with easier problems and gradually increase difficulty. This helps build confidence and understanding before tackling more complex tasks.</p> <p>Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.</p>	<p>understanding and to reinforce the concept from multiple angles.</p> <p>Scaffolded Problems: Start with easier problems and gradually increase difficulty. This helps build confidence and understanding before tackling more complex tasks.</p> <p>Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.</p>	<p>understanding and to reinforce the concept from multiple angles.</p> <p>Scaffolded Problems: Start with easier problems and gradually increase difficulty. This helps build confidence and understanding before tackling more complex tasks.</p> <p>Extension Challenges: Include a few challenging problems that encourage critical thinking and exploration beyond the basic concepts.</p>	
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