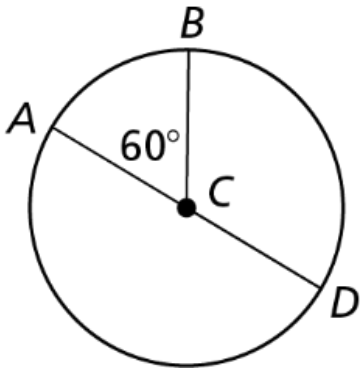
Lesson 11.1
Circumference and Arc Length

WARM-UP

Find the measure of the arc.

$\overset{\frown}{BD}$



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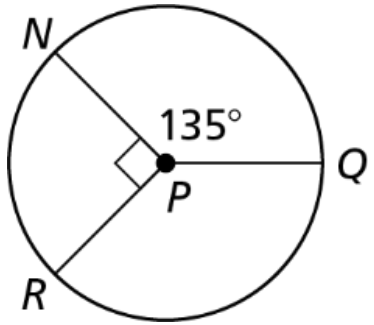


Learning Target:

Understand circumference, arc length, and radian measure.

Success Criteria:

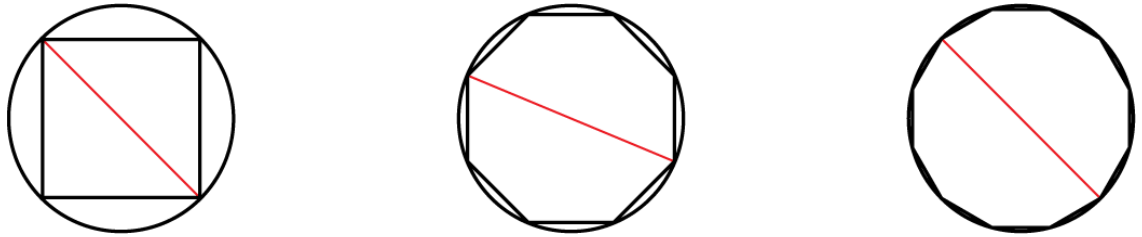
- I can use the formula for the circumference of a circle to find measures.
- I can find arc lengths and use arc lengths to find measures.
- I can solve real-life problems involving circumference.
- I can explain radian measure and convert between degree and radian measure.

Vocabulary

circumference, *p. 582*
arc length, *p. 583*
radian, *p. 585*

Using the Formula for Circumference

The **circumference** of a circle is the distance around the circle. Consider a regular polygon inscribed in a circle. As the number of sides increases, the polygon approximates the circle, and the ratio of the perimeter of the polygon to the diameter of the circle approaches $\pi \approx 3.14159\ldots$.

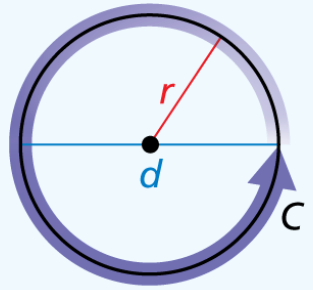


For all circles, the ratio of the circumference C to the diameter d is the same. This ratio is $\frac{C}{d} = \pi$. Solving for C yields the formula for the circumference of a circle, $C = \pi d$. Because $d = 2r$, where r is the radius, you can also write the formula as $C = \pi(2r) = 2\pi r$.

KEY IDEA

Circumference of a Circle

The circumference C of a circle is $C = \pi d$ or $C = 2\pi r$, where d is the diameter of the circle and r is the radius of the circle.



$C = \pi d = 2\pi r$

EXAMPLE 1 Using the Formula for Circumference

Find each indicated measure.

a. circumference of a circle with a radius of 9 centimeters

b. circumference of a circle with a radius of 26 meters

SOLUTION

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. Find the circumference of a circle with a diameter of 5 inches.

2. Find the circumference of a circle with a radius of 17 feet.

KEY IDEA

Using the Formula for Circumference

STUDY TIP
Just as the terms *point*, *line*, and *plane* are undefined, the distance around a circular arc is another example of an undefined geometric term.

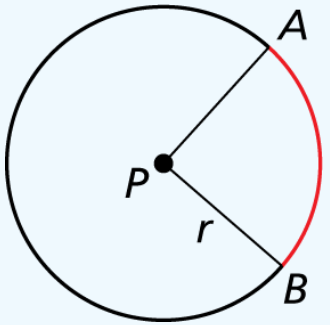
An **arc length** is a portion of the circumference of a circle. You can use the measure of the arc (in degrees) to find its length (in linear units).

Arc Length

In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

$$\frac{\text{Arc length of } \overset{\frown}{AB}}{2\pi r} = \frac{m\overset{\frown}{AB}}{360^\circ}, \text{ or}$$

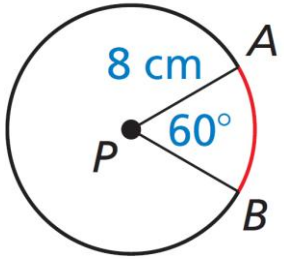
$$\text{Arc length of } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360^\circ} \cdot 2\pi r$$



EXAMPLE 2 Finding and Using Arc Lengths

Find each indicated measure.

a. arc length of $\overset{\frown}{AB}$



SOLUTION

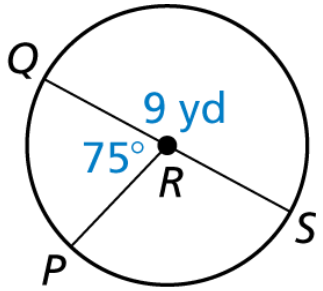
SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

3. Describe the difference between an arc measure and an arc length.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the indicated measure.

4. arc length of $\overset{\frown}{PQ}$



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KEY IDEA

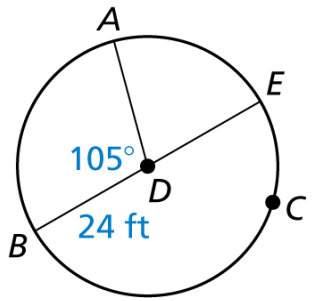
Converting between Degrees and Radians

Degrees to radians Multiply degree measure by $\frac{2\pi \text{ radians}}{360°}$, or $\frac{\pi \text{ radians}}{180°}$.

Radians to degrees Multiply radian measure by $\frac{360°}{2\pi \text{ radians}}$, or $\frac{180°}{\pi \text{ radians}}$.

Mini-Assessment

In Exercises 1 and 2, find the indicated measure.



1. circumference of $\odot D$

2. arc length of $\overset{\frown}{AB}$

3. You ride your bicycle 42 feet. The diameter of each wheel on your bicycle is 18 inches. How many complete revolutions does the front wheel make?

4. Convert 210° to radians.

5. Convert $\frac{5\pi}{12}$ radians to degrees.