

Educational Epiphany™

Districtwide PLC Protocol for Mathematics

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| Teacher/Teacher Team: Mr. Samuel, Mr. Bacchus, Ms. Wallace |
| Grade/Course: Geometry |
| Date: Week of September 4, 2023 |

| # | Planning Question | |
|---|---|--|
| 1 | Which state standard is your lesson progression addressing? | <p style="text-align: center;">Lesson 1.5 – Measuring and Constructing Angles</p> <p>G.CO.D.11 Perform formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).</p> <p>Foundational Standards: 7.G.A.2</p> |
| 2 | What mathematical concepts are embedded in the state standard? | <ul style="list-style-type: none"> • Bisect an angle using a compass. • Construct perpendicular lines, including the perpendicular bisector of a line segment. • Construct a line parallel to a given line through a point not on the line. • Use the virtual compass and line tool in dynamic geometry software to construct various geometric objects. • Develop methods using a variety of appropriate tools (compass, straightedge, string, reflective device, paper folding, etc.) to perform precise geometric constructions. • Explain informally why and how these construction methods work. <p>Understand the importance of precision in these constructions and attend to precision when performing geometric constructions.</p> |
| 3 | What teacher knowledge, reminders, and misconceptions are assumed in the standard? | <p>Knowledge:</p> <ul style="list-style-type: none"> • Students must be allowed to experiment with the construction tools to develop their own method to perform these constructions rather than just be given specific instructions to follow. They will need a basic understanding of the expected outcome. • It is through the process of the construction and particularly discovering the method that students will develop a deeper understanding of the properties of these objects. • Students will want to use a ruler to bisect a line segment or a protractor to bisect an angle, but when performing these formal constructions, students should not use tools that measure. Instead, they need to focus on the properties of the figures in the construction. Likewise, when students are using dynamic geometry software, they should avoid using automatic commands for bisecting and performing other constructions and use the virtual compass and line tool instead. • Requiring students to perform constructions by hand will help them discover the need for precision, which is essential in performing these constructions or they will not work. For example, a perpendicular bisector construction may not end up exactly in the middle or exactly perpendicular if the student does not use the same holes in the compass during the construction. Dynamic geometry software may help students perform the constructions precisely, particularly for students who struggle with using the tools precisely, but it is important that students also experience performing constructions by hand. • Developing the process of the methods leads to a deeper understanding of why and how each method works. Therefore, it is important that students be required to show their understanding by informally explaining what their chosen method does and why it works. <p>Reminders:</p> <ul style="list-style-type: none"> • In grade 7 (7.G.A cluster), students begin to experiment with mathematical tools to construct geometric figures and explore their relationships. In this course, students learn to use these and additional tools to perform constructions to explore and demonstrate geometric properties and help students visualize geometric theorems. |

Additional supporting and prerequisites standards are indicated on the curriculum map. In addition, this is not a comprehensive breakdown of each lesson for this weekly PLC protocol guide.

| | | |
|---|---|---|
| | | <ul style="list-style-type: none"> It is important that students understand that constructions serve a purpose. Therefore, pairing this standard with others throughout this course, including G.CO.A.3 and G.CO.D.12, will help students see the why behind these valuable skills. <p>Misconceptions:</p> <ul style="list-style-type: none"> Students frequently want to resort to using a ruler and protractor. The teacher needs to make the constraints for use of a particular tool clear. If students are not precise in a construction, it may not appear to work. The teacher needs to emphasize the importance of precision. Alternatively, using dynamic geometry software could alleviate some of these difficulties. |
| 4 | What objective(s) must be taught? In what order? Why? | <p>PBO:</p> <ul style="list-style-type: none"> SWBAT use a variety of tools and methods (compass, straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.) IOT perform formal geometric constructions. <p>Lesson objectives:</p> <ul style="list-style-type: none"> I can measure and classify angles. I can construct congruent angles. I can find angle measures. I can construct an angle bisector. |
| 5 | What academic language must be taught before the teacher models for students? How will the academic language be taught and assessed ? | <p>Academic Language:</p> <ul style="list-style-type: none"> Use – take, hold, or apply Variety – more than one; several Method – a step of a procedure of an experiment Compass – a tool used for drawing and drafting to create arcs, circles or other geometric figures Perform – carry out, accomplish, or fulfill Formal – characterized by precise respect for form Geometric – related to geometry Construction – a geometric figure made with only a straightedge and compass. <p>Instructional Practice 2: Strategies used to teach unfamiliar words will include:</p> <ul style="list-style-type: none"> 30 – 30 – 30 (common math-related word parts in the text, problem or objective) Point of Use Annotation of the Performance Based Objective Universal Language of Literacy Word and Definition Walls Word Parts Context Clues Point of Use Annotation of the Text (in Real Time) |
| 6 | What activities/practice problems are you planning to use for Launch the Lesson, Explore It, Examples & Self-Assessment, and Practice portions of the lesson? What did you learn from working the problems in advance of | <p><u>Technology Integration Suggestions: Big Ideas Platform</u></p> <ul style="list-style-type: none"> Dynamic Classroom Resources: Digital Example Videos Resources: Everyday Connections Video Series Lesson Example PowerPoints Resources: Explorations (Dynamic) <p>For technology integration resources and suggestions, please click here.</p> |

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using them in class with students?

Monday 09/04/2023

Labor Day (Break)

Tuesday 08/30/2023

Do Now 09/05/2023 (5 minutes)

Name: _____ **Period** ____

Point M is on \overline{PQ} , \overline{PM} has a length of 7 units, and Point N is the midpoint of \overline{MQ} .



If the length of \overline{NQ} is 4 units, how long is \overline{PQ} ?

- 3 units
- 11 units
- 15 units
- 18 units

Agenda

- Measure Angles
- Construct Angles
- Describe Angles

PBO

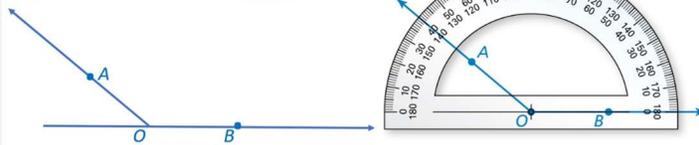
- 30 – 30 – 30 (common math-related word parts in the text, problem, or objective)
- Point of Use Annotation of the Performance Based Objective
- Universal Language of Literacy
- Word and Definition Walls

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Laurie's Notes

Launch the Lesson

? "What is the measure of $\angle AOB$?"



? "How can you find the measure of the angle?" Measure the angle using a protractor.

IMPORTANT How is a protractor used to measure an angle?

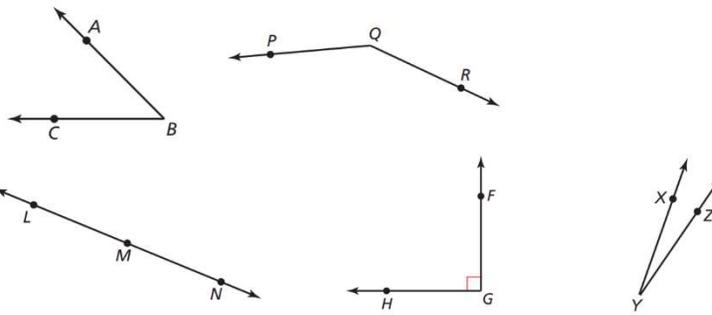
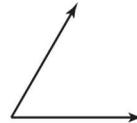
1.5 Measuring and Constructing Angles

EXPLORE IT! Analyzing a Geometric Figure

? "Look around the room. Where do you see examples of acute angles? right angles? obtuse angles? straight angles?"

Work with a partner.

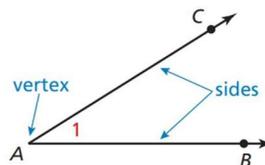
- Identify the figure shown at the right. Then define it in your own words.
- Label and name the figure. Then compare your results with those of your classmates.
- How can you *measure* the figure?
- Describe each angle below. How would you group these angles? Explain.



Naming Angles

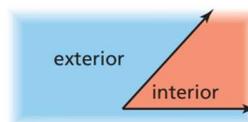
An **angle** is a set of points consisting of two different rays that have the same endpoint, called the **vertex**. The rays are the **sides** of the angle.

You can name an angle in several different ways. The symbol \angle represents an angle.



- Use its vertex, such as $\angle A$.
- Use a point on each ray and the vertex, such as $\angle BAC$ or $\angle CAB$.
Make sure the vertex is the middle letter.
- Use a number, such as $\angle 1$.

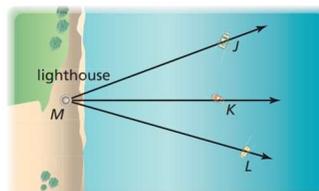
The region that contains all the points between the sides of the angle is the **interior of the angle**. The region that contains all the points outside the angle is the **exterior of the angle**.



EXAMPLE 1 Naming Angles



A lighthouse keeper measures the angles formed by the lighthouse at point M and three boats. Name three angles shown in the diagram.



SOLUTION

- $\angle JMK$ or $\angle KMJ$
- $\angle KML$ or $\angle LMK$
- $\angle JML$ or $\angle LMJ$

Measuring and Classifying Angles

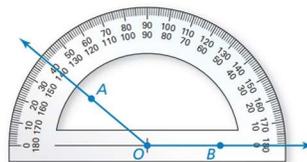
A protractor helps you approximate the *measure* of an angle. The measure is usually given in *degrees*.

POSTULATE

1.3 Protractor Postulate

Consider \overline{OB} and a point A on one side of \overline{OB} . The rays of the form \overrightarrow{OA} can be matched one to one with the real numbers from 0 to 180.

The **measure** of $\angle AOB$, which can be written as $m\angle AOB$, is equal to the absolute value of the difference between the real numbers matched with \overrightarrow{OA} and \overrightarrow{OB} on a protractor.

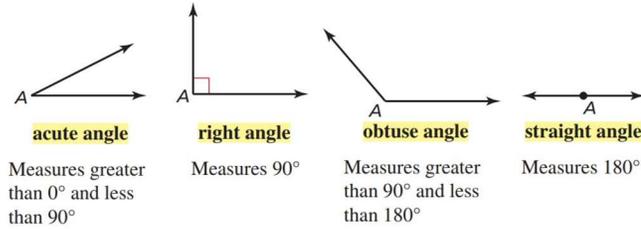


You can classify angles according to their measures.



KEY IDEA

Types of Angles

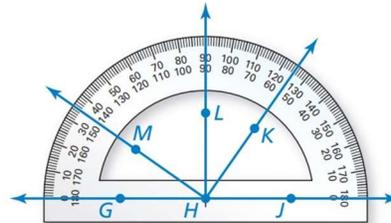


EXAMPLE 2 Measuring and Classifying Angles



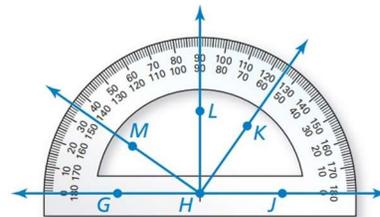
Find the measure of each angle.
Then classify the angle.

- $\angle GHK$
- $\angle JHL$
- $\angle LHK$



SOLUTION

- \overrightarrow{HG} lines up with 0° on the outer scale of the protractor. \overrightarrow{HK} passes through 125° on the outer scale. So, $m\angle GHK = 125^\circ$. It is an *obtuse* angle.
- \overrightarrow{HJ} lines up with 0° on the inner scale of the protractor. \overrightarrow{HL} passes through 90° . So, $m\angle JHL = 90^\circ$. It is a *right* angle.

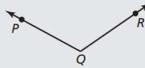


- \overrightarrow{HL} passes through 90° . \overrightarrow{HK} passes through 55° on the inner scale. So, $m\angle LHK = |90 - 55| = 35^\circ$. It is an *acute* angle.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Write three names for the angle.

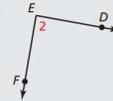
1.



2.



3.



4. **WHICH ONE DOESN'T BELONG?** Which angle name does *not* belong with the other three? Explain your reasoning.

$\angle BCA$

$\angle BAC$

$\angle 1$

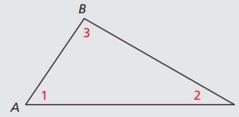
$\angle CAB$

Use the diagram in Example 2 to find the measure of the angle. Then classify the angle.

5. $\angle JHM$

6. $\angle MHK$

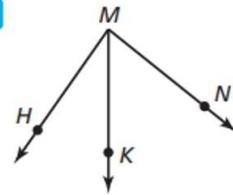
7. $\angle MHL$



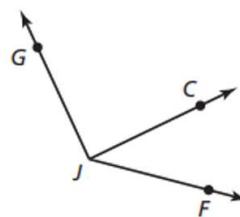
Exit Ticket

In Exercises 5 and 6, name three different angles in the diagram. [▶ Example 1](#)

5.



6.



Wednesday 09/06/2023

Do Now 09/06/2023 (5 minutes)

Name: _____ Period _____

1. Solve the equation.

$$x + 40 = 110$$

2. Solve the equation.

$$y - 55 = 35$$

Agenda

- Identifying congruent angles
- Copying an angle
- Using Angle Addition Postulate
- Finding angle measures

PBO

- 30 – 30 – 30 (common math-related word parts in the text, problem, or objective)
- Point of Use Annotation of the Performance Based Objective
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Identifying Congruent Angles

You can use a compass and straightedge to construct an angle that has the same measure as a given angle.

CONSTRUCTION

Copying an Angle

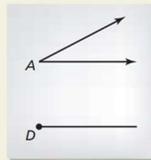


Use a compass and straightedge to construct an angle that has the same measure as $\angle A$. In this construction, the *center* of an arc is the point where the compass point rests. The *radius* of an arc is the distance from the center of the arc to a point on the arc drawn by the compass.

Copying $\angle A$.

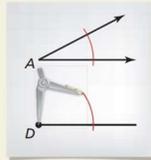
Solution

Step 1



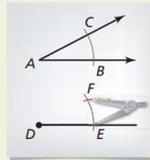
Draw a segment Draw an angle such as $\angle A$, as shown. Then draw a segment. Label point D on the segment.

Step 2



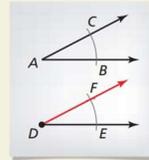
Draw arcs Draw an arc with center A . Using the same radius, draw an arc with center D .

Step 3



Draw an arc Label B , C , and E . Draw an arc with radius BC and center E . Label the intersection F .

Step 4



Draw a ray Draw \overrightarrow{DF} . $\angle D$ has the same measure as $\angle A$.

Two angles are **congruent angles** when they have the same measure. In the construction above, $\angle A$ and $\angle D$ are congruent angles. So,

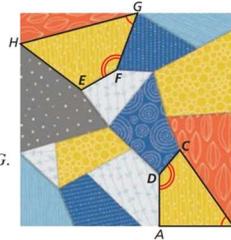
$$m\angle A = m\angle D \quad \text{The measure of angle A is equal to the measure of angle D.}$$

and

$$\angle A \cong \angle D. \quad \text{Angle A is congruent to angle D.}$$

EXAMPLE 3 Identifying Congruent Angles

- Identify the congruent angles labeled in the quilt design.
- $m\angle ADC = 140^\circ$. What is $m\angle EFG$?

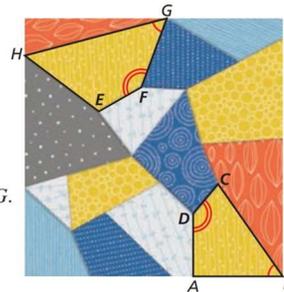


SOLUTION

- There are two pairs of congruent angles:
 $\angle ABC \cong \angle FGH$ and $\angle ADC \cong \angle EFG$.
- Because $\angle ADC \cong \angle EFG$,
 $m\angle ADC = m\angle EFG$.
 So, $m\angle EFG = 140^\circ$.

EXAMPLE 3 Identifying Congruent Angles

- Identify the congruent angles labeled in the quilt design.
- $m\angle ADC = 140^\circ$. What is $m\angle EFG$?



SOLUTION

- There are two pairs of congruent angles:
 $\angle ABC \cong \angle FGH$ and $\angle ADC \cong \angle EFG$.
- Because $\angle ADC \cong \angle EFG$,
 $m\angle ADC = m\angle EFG$.
 So, $m\angle EFG = 140^\circ$.

EXAMPLE 4 Finding Angle Measures

Given that $m\angle LKN = 145^\circ$, find $m\angle LKM$ and $m\angle MKN$.

SOLUTION

Step 1 Write and solve an equation to find the value of x .

$$\begin{aligned}
 m\angle LKN &= m\angle LKM + m\angle MKN && \text{Angle Addition Postulate} \\
 145^\circ &= (2x + 10)^\circ + (4x - 3)^\circ && \text{Substitute angle measures.} \\
 145 &= 6x + 7 && \text{Combine like terms.} \\
 138 &= 6x && \text{Subtract 7 from each side.} \\
 23 &= x && \text{Divide each side by 6.}
 \end{aligned}$$

Step 2 Evaluate the given expressions when $x = 23$.

$$\begin{aligned}
 m\angle LKM &= (2x + 10)^\circ = (2 \cdot 23 + 10)^\circ = 56^\circ \\
 m\angle MKN &= (4x - 3)^\circ = (4 \cdot 23 - 3)^\circ = 89^\circ
 \end{aligned}$$

► So, $m\angle LKM = 56^\circ$ and $m\angle MKN = 89^\circ$.

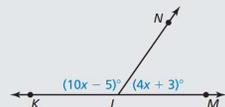
SELF-ASSESSMENT

- 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

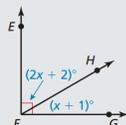
8. Without measuring, determine whether $\angle DAB$ and $\angle FEH$ in Example 3 appear to be congruent. Explain your reasoning. Use a protractor to verify your answer.

Find the indicated angle measures.

9. Given that $\angle KLM$ is a straight angle, find $m\angle KLN$ and $m\angle NLM$.

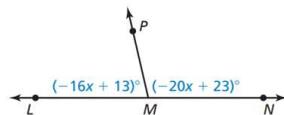


10. Given that $\angle EFG$ is a right angle, find $m\angle EFH$ and $m\angle HFG$.

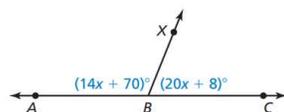


1.5 Practice WITH CalcChat AND CalcView

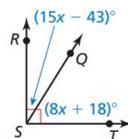
25. $\angle LMN$ is a straight angle. Find $m\angle LMP$ and $m\angle NMP$.



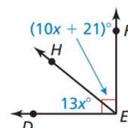
26. $\angle ABC$ is a straight angle. Find $m\angle ABX$ and $m\angle CBX$.



27. Find $m\angle RSQ$ and $m\angle TSQ$.

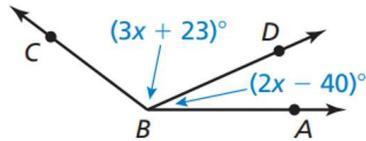


28. Find $m\angle DEH$ and $m\angle FEH$.



Closure

Display the diagram and say, "Given that $m\angle ABC = 143^\circ$, find $m\angle ABD$ and $m\angle DBC$."



Thursday 09/07/2023

Do Now 09/07/2023 (5 minutes)

Name: _____ **Period** ____

The point $M(-1, -2)$ is the midpoint of \overline{AB} . If point B has the coordinates $(4, 4)$, what is the coordinate of the endpoint A ?

- (-5, -6)
- (5, 6)
- (3, 2)
- (-6, -8)

Agenda

- Describing pairs of angles
- Identifying pairs of angles
- Using complementary and supplementary angles

PBO

- 30 – 30 – 30 (common math-related word parts in the text, problem, or objective)
- Point of Use Annotation of the Performance Based Objective
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1.6 Describing Pairs of Angles

Laurie's Notes

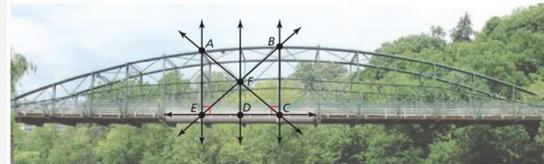
Launch the Lesson Aerial View of an Airport



Make a list of pairs of angles and their relationships.

EXPLORE IT! Identifying Pairs of Angles

Work with a partner. The Blackfriars Street Bridge in London, Ontario, Canada, is a bowstring arch-truss bridge. Use the diagram to complete parts (a)–(c).



A bowstring arch-truss bridge is one of the rarest types of bridges. The bridge above was built in 1875. There are few bridges of this type remaining today.

Identify a pair of the indicated angles. Do not use the same pair of angles twice.

- complementary angles
- supplementary angles
- adjacent angles
- vertical angles

Using Complementary and Supplementary Angles

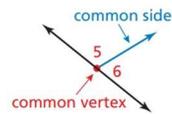
Pairs of angles can have special relationships. The measurements of the angles or the positions of the angles in the pair determine the relationship.



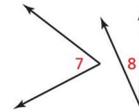
KEY IDEAS

Adjacent Angles

Adjacent angles are two angles that share a common vertex and side, but have no common interior points.

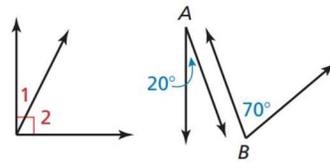


$\angle 5$ and $\angle 6$ are adjacent angles.



$\angle 7$ and $\angle 8$ are nonadjacent angles.

Complementary and Supplementary Angles

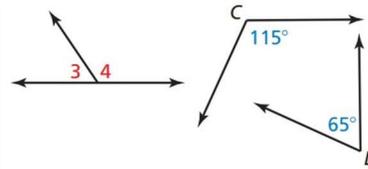


$\angle 1$ and $\angle 2$

$\angle A$ and $\angle B$

complementary angles

Complementary angles are two positive angles whose measures have a sum of 90° . Each angle is the *complement* of the other.



$\angle 3$ and $\angle 4$

$\angle C$ and $\angle D$

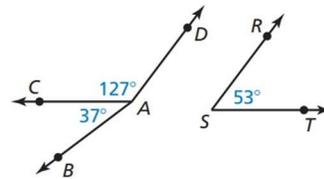
supplementary angles

Supplementary angles are two positive angles whose measures have a sum of 180° . Each angle is the *supplement* of the other.

EXAMPLE 1 Identifying Pairs of Angles



In the diagram, name a pair of adjacent angles, a pair of complementary angles, and a pair of supplementary angles.



SOLUTION

$\angle BAC$ and $\angle CAD$ share a common vertex and side, but have no common interior points. So, they are **adjacent angles**.

Because $37^\circ + 53^\circ = 90^\circ$, $\angle BAC$ and $\angle RST$ are **complementary angles**.

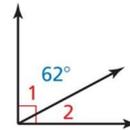
Because $127^\circ + 53^\circ = 180^\circ$, $\angle CAD$ and $\angle RST$ are **supplementary angles**.

EXAMPLE 2 Finding Angle Measures

- a. $\angle 1$ is a complement of $\angle 2$, and $m\angle 1 = 62^\circ$. Find $m\angle 2$.
b. $\angle 3$ is a supplement of $\angle 4$, and $m\angle 4 = 47^\circ$. Find $m\angle 3$.

SOLUTION

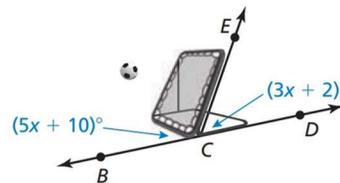
- a. Draw a diagram with complementary adjacent angles to illustrate the relationship.



- b. Draw a diagram with supplementary adjacent angles to illustrate the relationship.

**EXAMPLE 3** Modeling Real Life

When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCE$ and $m\angle ECD$.

**SOLUTION**

Step 1 Use the fact that the sum of the measures of supplementary angles is 180° .

$$m\angle BCE + m\angle ECD = 180^\circ$$

Write an equation.

$$(5x + 10)^\circ + (3x + 2)^\circ = 180^\circ$$

Substitute angle measures.

$$8x + 12 = 180$$

Combine like terms.

$$x = 21$$

Solve for x .

Step 2 Evaluate the given expressions when $x = 21$.

$$m\angle BCE = (5x + 10)^\circ = (5 \cdot 21 + 10)^\circ = 115^\circ$$

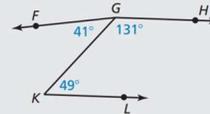
$$m\angle ECD = (3x + 2)^\circ = (3 \cdot 21 + 2)^\circ = 65^\circ$$

► So, $m\angle BCE = 115^\circ$ and $m\angle ECD = 65^\circ$.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

In Exercises 1 and 2, use the diagram.

1. Name a pair of adjacent angles, a pair of complementary angles, and a pair of supplementary angles.
2. Are $\angle KGH$ and $\angle LKG$ adjacent angles? Are $\angle FGK$ and $\angle FGH$ adjacent angles? Explain.
3. $\angle 1$ is a complement of $\angle 2$, and $m\angle 2 = 5^\circ$. Find $m\angle 1$.
4. $\angle 3$ is a supplement of $\angle 4$, and $m\angle 3 = 148^\circ$. Find $m\angle 4$.
5. $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles when $m\angle LMN = (4x - 2)^\circ$ and $m\angle PQR = (9x + 1)^\circ$.



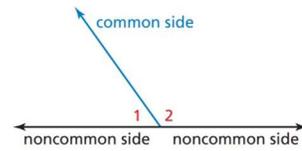
Using Other Angle Pairs



KEY IDEAS

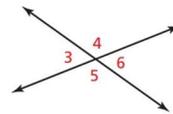
Linear Pairs and Vertical Angles

Two adjacent angles are a **linear pair** when their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.



$\angle 1$ and $\angle 2$ are a linear pair.

Two angles are **vertical angles** when their sides form two pairs of opposite rays.



$\angle 3$ and $\angle 6$ are vertical angles.

$\angle 4$ and $\angle 5$ are vertical angles.

EXAMPLE 4 Identifying Angle Pairs

Identify all the linear pairs and all the vertical angles in the diagram.

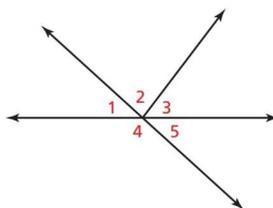
SOLUTION

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

- ▶ $\angle 1$ and $\angle 4$ are a linear pair.
- $\angle 4$ and $\angle 5$ are also a linear pair.

To find vertical angles, look for pairs of opposite rays.

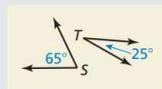
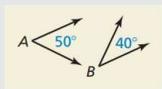
- ▶ $\angle 1$ and $\angle 5$ are vertical angles.



SELF-ASSESSMENT

- 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

6. **WRITING** Explain the difference between adjacent angles and vertical angles.
7. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.



Practice & Exit Ticket

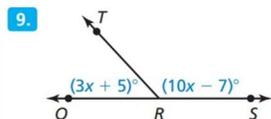
1.6 Practice WITH CalcChat® AND CalcView®

In Exercises 5–8, find the angle measure. ▶ Example 2

5. $\angle 1$ is a complement of $\angle 2$, and $m\angle 1 = 23^\circ$. Find $m\angle 2$.
6. $\angle 3$ is a complement of $\angle 4$, and $m\angle 3 = 46^\circ$. Find $m\angle 4$.
7. $\angle 5$ is a supplement of $\angle 6$, and $m\angle 5 = 78^\circ$. Find $m\angle 6$.
8. $\angle 7$ is a supplement of $\angle 8$, and $m\angle 7 = 109^\circ$. Find $m\angle 8$.

In Exercises 9–12, find the measure of each angle.

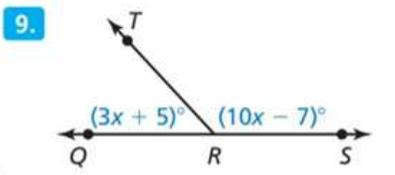
▶ Example 3



In Exercises 19–24, find the measure of each angle.

▶ Example 5

19. Two angles form a linear pair. The measure of one angle is twice the measure of the other angle.
20. Two angles form a linear pair. The measure of one angle is $\frac{1}{3}$ the measure of the other angle.
21. The measure of an angle is $\frac{1}{4}$ the measure of its complement.
22. The measure of an angle is nine times the measure of its complement.
23. The ratio of the measure of an angle to the measure of its complement is 4:5.
24. The ratio of the measure of an angle to the measure of its complement is 2:7.

| | | |
|---|---|---|
| | | <p>Exit Ticket Find the measure of each angle.</p>  <p>Homework Exercises 19 – 24.</p> <p>Friday 09/08/2023 Assessment – Lessons 1.3 – 1.6 Quiz – Microsoft Forms</p> |
| 7 | <p>What manipulatives might be integrated into the lesson? What did you learn from using the manipulatives in advance of using them in class with students?</p> | <p>Compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, protractor, etc.</p> <p>Reference: Interactive Manipulatives</p> <ul style="list-style-type: none"> • Didax Virtual Manipulatives |
| 8 | <p>What graphic organizer(s) might support students' conceptual understanding of the process outlined by the performance-based objective(s)?</p> | <p>Reference:</p> <ul style="list-style-type: none"> • Graphic Organizer Templates • Google Drawing Graphic Organizers Teacher Vision |

Additional supporting and prerequisites standards are indicated on the curriculum map. In addition, this is not a comprehensive breakdown of each lesson for this weekly PLC protocol guide.